2.13. Formal Semantics: Negations

The first, and simplest, of the molecular sentences are **negations**, built according to the second construction rule.

2. If ▲ is a formal sentence, then ~▲ is a formal sentence.

Like all the molecule-building rules, Rule 2 can ‘recycle’ its own output as input, constructing ever-bigger negations.

(and so on....)

4. ~ ~ ~P

3. ~ ~P

2. ~P

1. P

To keep up, the semantic rule must ‘recycle’ in the same way, matching negation construction step for step. Just as “~P” is constructed from “P,” so the truth table for “~P” is built from the truth table for “P”. The truth table just repeats the construction tree, *horizontally*.

P	**~P**

With just one sentence letter, two valuations cover all the possibilities.

P	**~P**
1	
0	

To determine what "~P" should be in each valuation, an English negation serves as our guide. So suppose "P" translates the sentence "It's raining," and "~P" means "It's not raining". Then the first valuation is a situation where it is raining. There the sentence "It's not raining" will be **false**.

P	**~P**
1	**0**
0	

The second valuation is a situation where "It's raining" is false – the very sort of situation where "It's not raining" will be **true**.

P	**~P**
1	0
0	**1**

That's in agreement with both English and common sense: the denial of a true sentence says something false, while the denial of a false sentence says something true. And that's so for any negation, not just "P". So we state the semantics of negations in full generality.

Negation Rule

▲	~▲
1	0
0	1

To appreciate this rule's power to recycle sentences, consider a more complex negation, "~~P".

3. ~ ~P

2. ~P

1. P

Once again the truth table follows the construction: just as “~~P” was built from “~P,” so the truth table for “~P” now serves as input, yielding a truth table for “~~P” as output.

P	**~P**	**~ ~P**
1	**0**	
0	**1**	

In the first valuation, where “~P” is false, its negation will be **true**.

Negation Rule

▲	**~▲**
1	0
0	**1**

P	**~P**	**~ ~P**
1	**0**	**1**
0	1	

And in the second valuation, where “~P” is true, its negation is **false**.

Negation Rule

▲	**~▲**
1	**0**
0	1

P	**~P**	**~ ~P**
1	0	1
0	**1**	**0**

As with construction, the semantics doesn’t need one rule for single negations, a second for double negations, etc. – just one *recycling* rule.

This example illustrates a further semantic concept as well. Note that "P" and "~~P" **have the same truth tables**.

P	**~P**	**~ ~P**
1	0	**1**
0	1	**0**

Sentences with the same truth tables are said to be **logically equivalent**.[1]

Reading "P" again as "It's raining," "~~P" will read in English as "It's not not raining" – or some prettier variation such as "It is not failing to rain." Now as a matter of fact English speakers take these two sentences to be making the very same claim – to **mean the same thing**.

That illustrates a point we'll have occasion to note again later: **logical equivalence** turns out to be a good test of when two sentences (logically) **mean the same thing**.

And that observation retires an outstanding debt left over from our treatment of translation. Recall that our preferred method of translation has us mechanically replacing each English form phrase with its matching formal connective, without reflecting on the meaning of the larger sentence – what we called the "x-ray" translation method.[2]

That approach left a lingering worry that by translating English sentences with the *same meaning* into *different* formal sentences, we obscure their semantic sameness. A prime example is the two sentences just discussed: "It's raining" and "It's not failing to rain". While English speakers tell us they share the same meaning, the second sentence, with two more negation phrases, translates as a formal sentence with two more tildes.

P: It's raining

It's raining	P
It's not failing to rain	~~P

[1] This is sometimes called "truth-functional equivalence".
[2] In 2.6.

The worry was that in so doing the ‘x-ray’ translation approach throws away a significant piece of information about sameness of meaning.

But now we see that the worry was baseless. While facts about sameness of meaning truly *aren’t* the business of the translation method, a faithful ‘x-ray’ translation into the formal language preserves those facts for later extraction by methods proper to the task: the semantic rules.

This observation makes good on an earlier promise as well. When first broaching the topic of semantics we noted its two faces: a theory of **truth-and-falsehood**, and also of **meaning**.[3] While we opted to pursue semantics along its truth-and-falsehood side, we now see that we’ve lost nothing in the choice. For without cost or effort this approach also yields a logical test of sameness-of-meaning in the bargain.

[3] In 2.11